Spatial Scale Problems and Geostatistical Solutions: A Review*

Peter M. Atkinson
University of Southampton
Nicholas J. Tate
University of Leicester

The concept of spatial scale is fundamental to geography, as are the problems of integrating data obtained at different scales. The availability of GIS has provided an appropriate environment to re-scale data prior to subsequent integration, but few tools with which to implement the re-scaling. This sparsity of appropriate tools arises primarily because the nature of the spatial variation of interest is often poorly understood and, specifically, the patterns of spatial dependence and error are unknown. Spatial dependence can be represented and modelled using geostatistical approaches providing a basis for the subsequent re-scaling of spatial data (e.g., via spatial interpolation). Geostatistical techniques can also be used to model the effects of re-scaling data through the geostatistical operation of regularization. Regularization provides a means by which to re-scale the statistics and functions that describe the data rather than the data themselves. These topics are reviewed in this paper and the importance of the spatial scale problems that remain is emphasized. **Key Words: geostatistics, re-scaling, sampling, scale.**

Introduction

Scale is a fundamental concept in geography, and it creates fundamental problems for geographers. Spatial scale provides a link between a property distributed in space and its representation, for example, as a map. Scale, therefore, provides key information on geographical models, and it is this link between scale and modelling which underlines the importance of scale to geography. Although the problems of representing and integrating data at different scales are well known, the continuing emphasis placed on scale-related issues in human geography (Amrhein and Wong 1996; Longley and Batty 1996) and physical geography (Quattrochi and Goodchild 1997) suggests that problems remain. The widespread availability of enabling technology such as GIS provides a platform for the integrated use of multiscale data, where "multiscale" refers to multiple scales of measurement (e.g., different spatial resolutions) and the potential for the user to re-scale (that is, to change the scale of measurement of) both the data and the model. Re-scaling requires not only the selection of appropriate models of multiscale spatial data variability—which in turn requires knowledge about the scaling characteristics of the property of interest—but also appropriate methodologies and tools to implement the re-scaling. As we shall show in this paper, geostatistical techniques, and to a lesser extent fractal techniques, can be used to address several problems of data and model scaling.

The term "scale" is unfortunately an ambiguous term often used to refer to both *the amount of detail* and the *spatial extent* of a geographic coverage (Goodchild and Proctor 1997). Traditionally, cartographic or map scale is reported as a ratio or representative fraction between a unit distance on the map and its equivalent distance on the ground. This definition of scale is strictly correct, but may lead to confusion, as 1:10,000 is a large scale in relation to 1:100,000 although the number 10,000 appears smaller than 100,000. A 1:10,000 scale map represents a larger scale than a 1:100,000 map because the values are ratios and 1/10,000 is a larger value than 1/100,000. This apparently minor issue is exacerbated when one considers the use of the word "scale" in everyday language. When one talks of a large-scale process, a large-scale phenomenon, or a large-scale investigation, one simply means a large process, phenomenon, or investigation. In these examples scale is used simply to mean *size* or *extent*, and is redundant. This contradicts the definition of cartographic scale above. For example, a map that covers the globe has a small cartographic scale, but an investigation that covers the globe is large-scale. The definition

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of scale synonymous with extent is used in disciplines such as physics, biology, and ecology. It is this simpler definition of scale which we adopt throughout this paper.

From the outset it is important to distinguish two kinds of spatial scale, both of which are important. The first is concerned with the *scales of spatial measurement*, and the second with the *scales of spatial variation* present in data that result from measurement. Both kinds of scale are of interest in this paper and are, therefore, treated fully in later sections. For the present we establish a conceptual framework which leads naturally to the distinction between scales of spatial measurement and scales of spatial variation. (Concentrating on the spatial component by no means implies that the temporal component of spatial variation is any less important, but rather is beyond the scope of this paper.) The framework, illustrated in Figure 1, is a simple one in which reality is always observed through a particular spatial sampling framework to produce spatial data. Two important observations follow from this. First, one can never observe "reality" independent of some sampling framework, so that what we observe is always a filtered version of reality. Even the data that we obtain directly through our senses (e.g., sight) represent a filtered version of reality subject to our perceptions. Second, since one can never observe reality independent of sampling, the scales of spatial variation alluded to earlier relate only to spatial *data*. Since spatial data are obtained through sampling with particular scales of measurement, the scales of variation observable in spatial data are inextricably linked to the scales of measurement through which they were obtained.

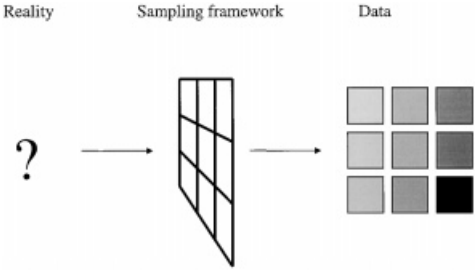

Figure 1: *Conceptual model involving reality, the sampling framework, and data (from Atkinson and Tate 1999, reproduced with permission).*

The conceptual framework outlined above is of fundamental importance and has wide-ranging implications for data acquisition and modelling. This paper reviews recent literature concerned with scale problems in geography. In addition, to draw the reader's attention to the geostatistical operation of regularization, a discussion is provided on the geostatistical techniques for re-scaling data and data models. In the second section, we consider the importance of spatial scale in geography, while in the third section we introduce scales of measurement. In the fourth section, we consider scales of variation in spatial data and the modelling of spatial variation, with a focus on models of second-order properties. Finally, in the fifth section, we discuss some approaches to changing the scale of measurement.

Spatial Scale in Geography

Within geography, spatial scales of inquiry range conservatively from 10^{-2} to 10^{6} m. Geographers have long been aware of the sensitivity of spatial data to the scale of measurement (e.g., Harvey 1969) and the utility of a multiscale approach to description (e.g., Stone 1972). However, the recent availability of tools such as GIS and enhanced computing power has facilitated the multivariable and multiscale analysis and integration of spatial data. This in turn has led to a focus on scale issues and concern with the nature of spatial variability in disciplines such as remote sensing (Woodcock and Strahler 1987; Quattrochi and Goodchild 1997), landscape ecology (Turner et al. 1989), geomorphology (Phillips 1988) and hydrology (Blöschl and Sivapalan 1995).

It is generally accepted that nearly all environmental processes are *scale-dependent* (e.g., Davis et al. 1991; Davis and Simonett 1998). In conjunction with a particular scale of measurement, this will influence the observed magnitude of spatial variation. Resultant patterns of spatial variation will often possess a *nested multiscale* (Burrough 1987, 1993) or *hierarchical* structure (Urban et al. 1986; King 1991; De Boer 1992). Different scales of measurement (e.g., different spatial resolutions) reveal variables to be homogenous at one scale yet heterogenous at another, regular at one scale yet irregular at another. Heterogeneity can also be increased simply by increasing the spatial ex-

tent of the study area (Haining 1989; King 1991). The presence of scale-dependent heterogeneity and irregularity is important in two main contexts: 1) defining a suitable scale range for a given investigation (e.g., a range of spatial resolutions which will reveal the variation of interest), and 2) characterizing the nature of the spatial variation so that the scales of measurement may be changed—that is, so that the data may be re-scaled.

Often it is desirable to focus on a particular scale of spatial variation that results from a specific process. In such circumstances, not all available scales of measurement need to be considered. It would be perfectly appropriate to represent topography as a smooth contour map at the drainage basin scale, but this would be inappropriate for examining sheet flow on an individual hillslope unit. De Boer (1992) noted that the practical choice of a scale range of interest is usually dictated in terms of a predetermined scale of the system under study and the overall objective of the investigation. In this context, certain scales will be important while others can effectively be ignored, since an implication of the existence of scale-dependent processes is that for sufficiently different measurement scales spatial patterns are independent (Phillips 1988). This leads to a fundamental question that has been raised in geomorphology (De Boer 1992), remote sensing, and GIS (Davis et al. 1991; Atkinson and Curran 1995): what are the optimal measurement scales for the investigation of a specific problem?

Often, there is a need for data sampled at one scale or location to be extended or generalized to other scales or locations and perhaps (e.g. in the context of a GIS) combined with other data. However, the existence of scale-dependent spatial variation makes the processes of data re-scaling and data integration problematic. In particular, the commonly employed techniques of averaging, smoothing, extrapolating, and interpolating to different scales of measurement are hazardous, particularly when combined with other problems such as replacing missing data (e.g., Bennett et al. 1984). In fact, the presence of scale-dependent heterogeneity has been recognized as providing a fundamental constraint on the comparison of multiscale phenomena in fields such as landscape ecology (e.g., Turner et al. 1989; Turner and Gardner 1991) and for the integration of remote sensing and GIS data (Quattrochi and Lam 1991).

Although a distinction has been made above between the choice of a scale range for a given analysis and the problems of practical data integration, they are in fact related elements of the same problem. Ideally, information on the *scaling properties* of environmental variables should act as a guide for both the acquisition of appropriate data and any subsequent multiscale and multivariable integration of such data. In practice, such information is often unavailable. This is partly because the specific pattern of *spatial dependence* (often referred to as autocorrelation) may be unknown or unique to a site or region, making it difficult to generalize (Davis et al. 1991). Further, within a nested hierarchical system, such as the ecological landscape, the dynamics and pattern at the landscape scale are often the result of interactions among lower-level systems (Urban et al. 1986; O'Neill et al. 1991), producing a scale differential problem sometimes referred to as the *dichotomy of scale* (Mark 1980) or simply as the *scale* or *scaling problem* (Harvey 1969; Jeffers 1988). Therefore, there is often a need to address processes and patterns at a variety of scales, using either a combined *top down*, *scaling down*, or *downscaling* approach analyzing spatial pattern at the landscape scale or a *bottom up*, *scaling up*, or *upscaling* approach assembling the landscape from individual finer spatial resolution process-based components (e.g., Urban et al. 1986; King 1991). The linkages across scales that are required for a multiscale approach are summarized in Figure 2. Indeed, in the context of landscape ecology, Milne (1991) suggested that a key to understanding heterogeneity was to conduct the analysis across a wide range of measurement scales and to extract parameters that are *robust* to changes in scale. In a similar context, King (1991) recognized two main challenges: 1) the definition of spatial heterogeneity, and 2) the correct integration and aggregation of data. As we show below, geostatistical techniques allow us not only to conduct analysis across a range of scales, but also to model spatial heterogeneity and re-scale both the data and the model to allow integration.

To illustrate the concepts addressed in the following sections we provide an example. The data used in the example are taken from Atkinson (1999) and Stein et al. (1999). These data

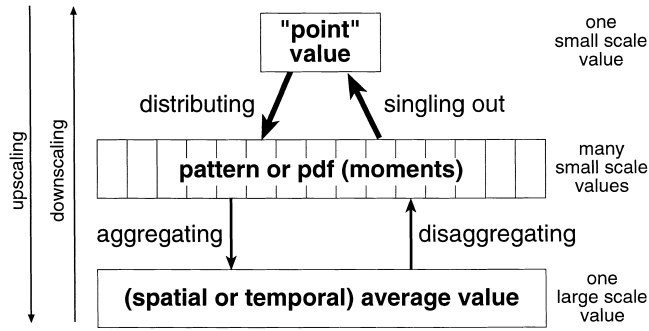

Figure 2: *Graphical representation of the processes of upscaling and downscaling (from Blöschl and Sivapalan 1995, reproduced with permission).*

are a Landsat Thematic Mapper (TM) image of part of the Netherlands. Details of the full Landsat TM image are given in the Stein et al. (1999) reference. The full image consists of seven wavebands, with each waveband consisting of 1,130 columns and 960 rows. The image covers Enschede just southeast of the centre, Hengelo just northwest of the centre, Gronau to the east, and Oldenzaal to the northeast.

One of the available seven wavebands in the red (0.63 μm–0.69 μm) wavelengths was selected for study to reduce redundancy between the multiple wavebands and to simplify the presentation. Only a small subset of the original image was considered for analysis: a 128 by 128 pixel subset covered entirely by the urban area of Enschede (Fig. 3). A histogram of the subset image, which is approximately Gaussian, is shown in Figure 4.

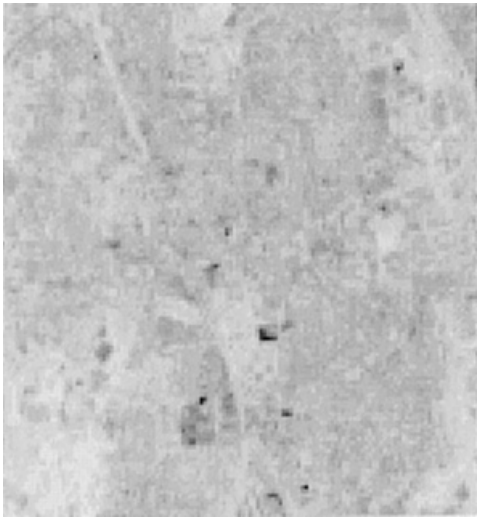

Figure 3: *Landsat TM red waveband subimage of Enschede, The Netherlands.*

Scales of Measurement

The scales of measurement are determined by the sampling framework (or strategy). The sampling framework can itself be divided into the spatial or geometrical characteristics of each individual *observation* and the spatial coverage of the *sample*.

A Single Observation

The size, geometry, and orientation of the space on which an observation is defined is known in geostatistics as the *support* (Matheron

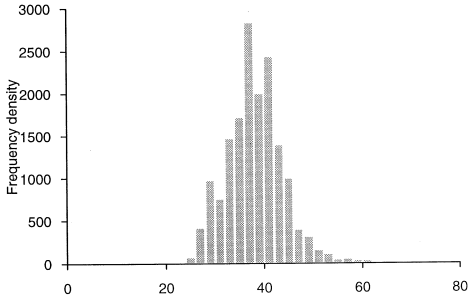

Figure 4: *Histogram of Landsat TM red waveband subimage (from Atkinson 1999, with kind permission of Kluwer Academic Publishers).*

1965). This concept is entirely general. Image pixels in data defined within the raster data model are a good example of a support. Similarly, zones used in the UK census of population, such as Enumeration Districts (EDs) and wards, represent supports for the single values (e.g., population count) often associated with them. In the former case the support is fixed, while for many kinds of census data the support is variable. The variable support encountered in census data leads to the well documented Modifiable Areal Unit Problem (MAUP) (Openshaw 1984; Amrhein and Wong 1996; Longley and Batty 1996). The MAUP is comprised of two components: an aggregation problem (central to the present paper) and a zonation problem. It arises because census data are defined for a support that varies from observation to observation. The size of support is similar to the spatial resolution of raster-based imagery except that the former relates to single observations while the latter is a function of multiple observations.

The support is an important concept because it is a fundamental scale of measurement (Moellering and Tobler 1972; Thornes 1973; Openshaw 1984). The effect of the support on the scales of natural variation that are detected has been demonstrated most notably in physical geography and, in particular, remote sensing (e.g., Clark 1990; Sèze and Rossow 1991) and in human geography through the MAUP (e.g., Openshaw 1984). To understand how the support, as a scale of measurement, affects the scales of spatial variation that are detectable from such measurement, it is necessary to examine how the support interacts with the underlying spatial variation to produce a single value.

In practice, values are derived through measurement over a support v, a finite element of space with a specific size, geometry, and orientation. Therefore, assuming a continuous random field, an observation $z_v(\mathbf{x}_0)$ may be treated as a realization of the random variable (RV), $Z_v(\mathbf{x}_0)$, which is the spatial mean or integral of Z over v centered on $\mathbf{x}_0$. Formally:

$$Z_v(\mathbf{x}_0) = \frac{1}{v} \int_{v(\mathbf{x}_0)} Z(\mathbf{y}) \mathrm{d}\mathbf{y} \qquad (1)$$

where $Z(\mathbf{y})$ is the property Z defined on a *punctual* (point) support. The important point is that the underlying spatial variation is averaged over the support to produce a *single mean value*. A continuous random field is assumed to keep notation consistent, but other models (e.g., models for point processes; Bailey and Gatrell 1995) could be treated in a similar way (e.g., census data are averages over the space represented by EDs, wards, and so on, even though the individuals could be treated as representing points).

The support may also vary in shape and orientation. For example, for remotely sensed imagery (e.g., the Landsat TM image of Enschede) the support may be approximated by a Gaussian or bell-shaped centre weighting function (known as the point-spread function) such that underlying spatial variation at the centre of the support receives more weight than that towards the edges. Where the support has an anisotropic shape—that is, the support varies with the angle of orientation—the orientation in which the support is placed may be important.

Spatial Sampling

The support is only one aspect of the spatial sampling framework. Three further important components of a sampling framework are the *sampling scheme*, *sampling density*, and *sample size*. The sampling scheme refers to the spatial pattern of the sample observations. Examples of sampling schemes are the random, stratified random, and systematic (including square grid and equilateral triangular) schemes. The sampling density refers to the number of observations per unit area. Finally, the sample size is the total number of observations. Collectively, the sampling scheme, sampling density, and sample size define the *spatial coverage*: the set of distance and direction vectors between the observations of the sample. For remotely sensed imagery, complete cover is provided; for a given area on the ground, the sampling framework is determined largely by the spatial resolution.

Note that two scales of measurement are identified. The first is the support and the second is the spatial coverage of the sample. The importance of these scales of measurement will become clear when spatial variation is examined in the context of spatial dependence.

Spatial Variation

In spatial statistics, variation in spatial data is described most simply through its first-order and

second-order properties. The first-order property of interest here is the mean, while the second-order properties of interest are the dispersion (or sample) variance $D^2(v,V)$ (representing the variation between all data v within V) and the covariance function $C(\mathbf{h})$ (representing the variation between pairs of data $[v,v_h]$ at specific distance and direction vectors of separation or lags $\mathbf{h}$). We can model a set of observations distributed spatially as a regionalised variable (ReV), a realization of a spatial set of RVs known as a random function (RF). In simple terms, a RF is a spatial set of cumulative distribution functions (cdfs) from which the sample data are assumed to be drawn. The first- and second-order properties relate to the RF from which the realization (sample) is supposed to have been drawn. That is, the mean and covariance are parameters of the RF model, not the data.

Stationarity

Where the mean and covariance are constant from place to place, the RF is said to be second-order stationary. Second-order stationarity is an important requirement of certain statistical techniques, which collectively are referred to as "time-series" analysis and which are often applied to spatial data (e.g., spectral analysis). This condition of second-order stationarity allows the pooling together of observations taken at different locations in space (Journel 1993). Unfortunately, samples of spatial data are often not stationary, and possess trends in the mean and a variance that increases with the extent of the region of study (Haining 1989). Where the mean is non-stationary and a trend is implied, it may be possible to remove this first-order effect by subtracting a trend surface from the data. However, since stationarity *sensu stricto* is a property of the RF and not the data (Myers 1989) it is untestable from sample data. For example, it is not possible to know whether the spatial variation observed in data is due to first- or second-order effects. Thus, the choice of RF model may be seen as somewhat arbitrary. Essentially, a model with a stationary mean and a large variance or a model with a non-stationary mean and a small variance could be chosen. Generally, we can detect suggestions that a non-stationarity model may be appropriate using various spatial exploratory data analysis (EDA) techniques such as the variogram cloud (Gunnink and Burrough 1996). With this constraint in mind, we concentrate on second-order properties in this paper.

The reason for concentrating on second-order properties is that a single datum is usually not informative. If we have a single datum with a given value (let us say 97), it contains strictly zero information on its own. We might perceive it as informative only if we have an *a priori* framework or context in which to understand the datum. Thus, if we know that the value represents the percentage of people who voted Liberal Democrat in the 1997 UK election, then we would recognize the value as large relative to the expected value. Without such a context, the value is not informative. That is, information exists in the relations *between* measurements and not in them. The same reasoning applies if the value represented a single elevation spot height: it is meaningless alone, and only informative in combination with other values. Since second-order properties are concerned with the relations between data, the second-order properties of spatial data hold a special significance.

Spatial Dependence and the Variogram

Spatial dependence is the likelihood that observations close in space are more alike than those further apart. This observation is generally known as Tobler's first law of geography (Tobler 1970; Goodchild 1987). Most geographic phenomena distributed spatially over the surface of the Earth are spatially dependent, at least at some scale. The ubiquity of spatial dependence in geographic variables at a variety of measurement scales has been detected in geomorphological data (Robert and Richards 1988; Oliver et al. 1989a,1988b; Bian and Walsh 1993), soil data (Webster 1985), ecological data (Rossi et al. 1992), remotely sensed images (Atkinson and Curran 1995), and epidemiological data (Oliver 1996). It is important to emphasize that spatial dependence is intuitively *necessary*; if properties were spatially independent and uncorrelated at all scales, all realizations (or observed values) at all places would be the same. There would be no form or structure, which would imply an absence of process.

Here we represent spatial dependence with a function known as the variogram (related to the covariance function by $\gamma(\mathbf{h}) = C(0) - C(\mathbf{h})$). The semivariance is defined as half the ex-

pected squared difference between the RFs $Z(\mathbf{x})$ and $Z(\mathbf{x} + \mathbf{h})$ at a particular lag $\mathbf{h}$ (Matheron 1965). The variogram $\gamma(\mathbf{h})$ defined as a parameter of the RF model is then given by:

$$\gamma(\mathbf{h}) = \frac{1}{2}\mathrm{E}[\{Z(\mathbf{x}) - Z(\mathbf{x} + \mathbf{h})\}^2] \qquad (2)$$

Developed as a tool for ore-reserve estimation, the variogram is based on a weaker assumption of stationarity than the covariance known as intrinsic stationarity of the RF: essentially, stationarity of the *differences* $Z(\mathbf{x}) - Z(\mathbf{x} + \mathbf{h})$, with the value of $\gamma(\mathbf{h})$ dependent only on $\mathbf{h}$. The variogram is in fact part of a wider body of statistical techniques known as geostatistics with which geographers are now familiar (Oliver et al. 1989a, 1989b). Geostatistics appears most often in quantitative geography in the form of a technique for estimation known as kriging, most commonly used for spatial interpolation (e.g., Oliver and Webster 1990). The sample (or experimental) variogram may be obtained from $P(\mathbf{h})$ pairs of sample data at lag $\mathbf{h}$ $\{z_v(\mathbf{x}_i), z_v(\mathbf{x}_i + \mathbf{h}), i = 1,2,3,\ldots, P(\mathbf{h})\}$ defined on supports v of size $|v|$ by:

$$\gamma_v(\mathbf{h}) = \frac{1}{2P(\mathbf{h})}\sum_{i=1}^{P(\mathbf{h})}\{z_v(\mathbf{x}_i) - z_v(\mathbf{x}_i + \mathbf{h})\}^2 \qquad (3)$$

Directional variograms for four different orientations were estimated for the Landsat TM red waveband image of Enschede (Fig. 5). The orientations are given on the figure. There

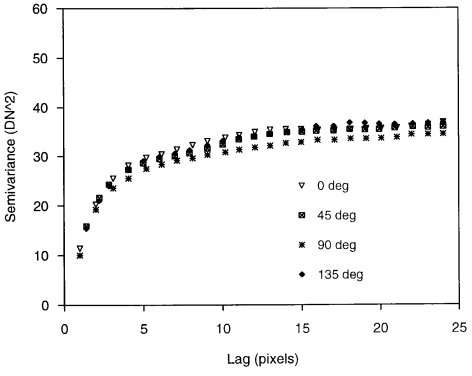

Figure 5: *Directional variograms of Landsat TM image in the red waveband at a pixel size of 30 m (from Atkinson 1999, with kind permission of Kluwer Academic Publishers).*

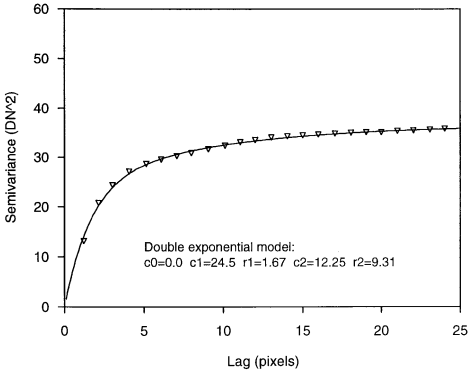

Figure 6: *Sample variogram of Landsat TM image in the red waveband at a pixel size of 30 m fitted with a nested exponential model (from Atkinson 1999, with kind permission of Kluwer Academic Publishers).*

is little variation between the variograms for different orientations. For this reason, it was considered appropriate to estimate an omnidirectional variogram representing all orientations simultaneously (Fig. 6, discrete symbols).

Modelling the Sample Variogram

For most geostatistical procedures, a mathematical model must be fitted to the sample variogram. The model must be conditional negative semi-definite (CNSD) to ensure that all possible linear combinations of the RF described cannot be negative, and for most applications the model is selected from several that are known to be CNSD or "authorized" for the dimensions of the space over which the RF is defined. Examples of the more common models expressed mathematically and graphically are presented by Webster and Oliver (1990), and five of them can be seen in Figure 7.

In most cases, the model fitted to a sample variogram approaches and intercepts the ordinate at some positive finite value known as the *nugget variance* c_0 (Fig. 7a). The nugget variance is due to unresolved variation including measurement error, but it may also arise from sampling uncertainty in estimating the variogram and uncertainty in model fitting. It has been suggested that where the observations are adjacent (as with the raster data model) or overlapping (as with remotely sensed imagery) the nugget variance may be used as an estimate of measurement error (Curran and Dungan 1989;

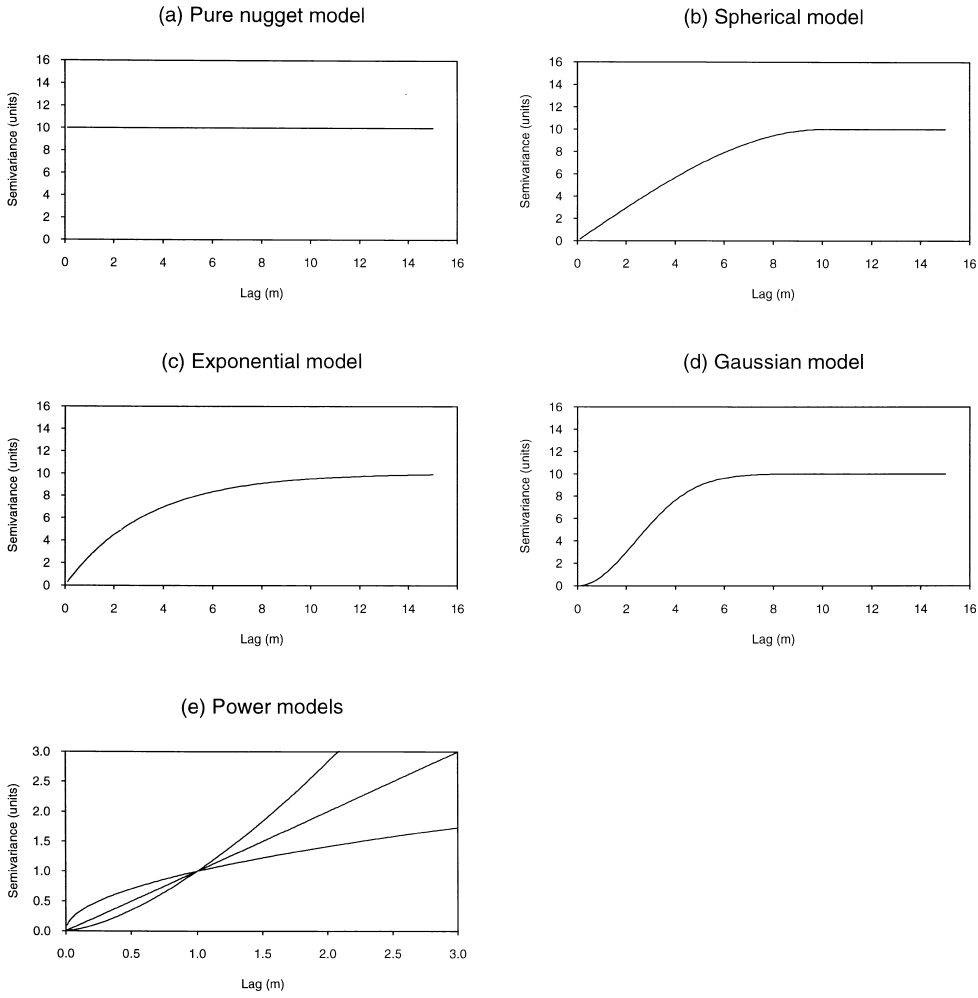

Figure 7: *Five variogram models represented graphically. Three power models are shown.*

Atkinson 1993). The nugget effect model is one in which the semivariance is constant for all lags (Fig. 7a) and on its own rarely provides a satisfactory fit to sample variograms.

In general, there are two types of model, referred to as unbounded and bounded, which may be fitted to the sample variogram. For unbounded models, the semivariance increases indefinitely with lag. The power model (Fig. 7e), of which the linear model (Fig. 7e, straight line) is one case, is an example of an unbounded model. For bounded models, the semivariance reaches a maximum known as the *sill*, at a fixed lag known as the *range a* (Fig. 7a–d). The sill represents the *a priori* variance $D^2(v,\infty)$, and this includes variation due to measurement error. The term c_1 (where $c_0 + c_1$ = sill) is referred to as the structured variance and represents spatial variation that is spatially dependent at some scale.

The range a and shape of the variogram model can be used to infer the scales of spatial variation present in the sample data. In particular, the range relates to the maximum scale of spatial variation, while further, "smaller" scales of spatial variation may be implied by the shape

of the model. For example, the exponential variogram model seen in Figure 7c is given by:

$$\gamma(h) = 1 - \exp(-h/r) \qquad \text{for } 0 < h \quad (4)$$
$$\gamma(h) = 0$$

where $h = |\mathbf{h}|$ for isotropic variation, and r is the distance parameter of the model such that an effective range a' may be computed as $a' = 3r$. This model implies a distribution of scales of spatial variation, as opposed to a single predominant scale of variation. Further, variogram models are additive, so that several authorized models may be added to provide a close approximation to the sample function. This implies that it is possible to have nested models, each with its own range representing a different scale of spatial variation (Webster and Oliver 1990). For example, the nugget variance of most fitted models is achieved by adding the nugget effect model to another type of model. The important point is that the variogram informs us about the scales of spatial variation in sample data.

The variogram of the Landsat TM image of Enschede was fitted with several authorized models by weighted least squares approximation using the Splus™ software. The double (nested) exponential model provided the best fit. The fitted model, together with the coefficient values, is shown in Figure 6.

Scale Independence and Fractals

The term fractal is a neologism coined by Mandelbrot (1967) to describe irregular patterns that possess no clear scale of variation. Although there are many types of fractals, at the most basic, a fractal object possesses a pattern which repeats either exactly or statistically as the scale of observation changes, a trait termed *self-similarity.* Further, the degree of irregularity of a fractal form is usually expressed in terms of fractal dimension D, which possesses a non-integer value in between the more familiar topological dimensions. For example, a fractal curve such as a coastline possesses a dimension in between the topological dimension $D_t = 1$ of a line and $D_t = 2$ of an area. The magnitude of D is determined by the degree to which the fractal object is more irregular or "space filling" than a simple straight line.

Fractals can be defined in terms of the variogram. The variogram models in Figure 7e are of a power law form, yet they also describe one-dimensional fractal or fractional Brownian functions of the form:

$$\gamma(h) = 0.5h^{(4-2D)} \quad (5)$$

Within geography, and particularly physical geography, there has been considerable effort expended on both the detection of fractal forms and the estimation of D using the variogram-based method (Chase 1992; Klinkenberg and Goodchild 1992; Bian and Walsh 1993; Burrough 1993). Indeed, fractal patterns have been detected in an increasing range of both physical and human geographical contexts (e.g., Lam and DeCola 1993; Batty and Longley 1994).

The importance of fractals to issues of scale cannot be understated, for two main reasons. First, the presence of fractals in geographic patterns would appear to imply an absence of scale-dependent processes. The incongruence of this with accepted process understanding has led some to speculate that spatial patterns are only fractal between certain size limits (Mark and Aronson 1984) and that fractals may be limited to a kind of null hypothesis model of the absence of scale-dependencies to test against reality (Goodchild 1988). Alternatively, the chaotic forms produced by non-linear processes also possess a fractal structure, and it is tempting to relate fractal patterns detected in geographical data to some chaotic process regime. Whether or not this is the case remains to be seen.

Second, and perhaps more important from the perspective of this paper, fractals are a model of underlying spatial variation which is *independent of the sampling framework.* If wished, fractal models could be used to simulate spatial variation, which could then be sampled with a given sampling framework to create data. All other models of spatial variation are fixed at a given scale of measurement. Similarly, in terms of the MAUP, if variables with a fractal spatial pattern can be identified, not only might the magnitude of fractal dimension provide useful information on the rate of change of scale, but these variables would be insensitive to re-scaling, and the choice of spatial units for the representation of data (Fotheringham 1990).

The pace of fractal research has slowed down in recent years, perhaps due to some controversy over how fractal dimensions should be computed (Klinkenberg and Goodchild 1992). Further, fractals have often been used only to model surface roughness (Xu et al. 1993) or for

simulation (Tate 1998). In spite of this, no other mathematical model has provided as realistic a description of the underlying spatial variation and forms found in nature (Barnsley 1989).

Changing the Scale of Measurement

In the previous two sections, scales of measurement and scales of spatial variation were considered. In this section, the emphasis is on *changing* the scale of measurement, and how different scales of measurement affect the scales of spatial variation which are observed.

Scaling the Data

It is possible to directly compare point measurements of one variable with area data (say, raster-based imagery) of another variable. The situation arises commonly in remote sensing, where the property to be estimated at the ground is measured on supports that are very small in relation to the pixels of the imagery. It is also increasingly common in GIS. However, the simple correlation coefficient between the two variables will be expected to be small where there is much variation within the larger supports. If the objective is to model the relation between the two variables (e.g., with regression) and use it to predict the point variable at unobserved locations, the accuracy of the predictions may be low. This strategy is not recommended and some attempt should be made to re-scale or "scale up" the point measurements before comparison.

Assume that the support of the point variable is 1 m and the area data are remotely sensed imagery with a support or spatial resolution of 1.1 km (by 1.1 km) (as for National Oceanographic and Atmospheric Administration [NOAA] Advanced Very High Resolution Radiometer [AVHRR] imagery). It is clear that complete coverage of the 1.1 km support (referred to as a ground resolution element or GRE) with 1 m observations would be near impossible (requiring over 1.2 m observations). Therefore, to estimate the mean value for each of several 1.1 km areas one should sample *within* the larger areas (Fig. 8).

Assume that the only data available are a random spatial sample of the property of interest distributed within the entire scene and defined on a support of 1 m. The problem then is that one does not have a spatial sample within each larger support, and therefore it is not possible to average the variation to estimate mean values. Therefore, alternative approaches including techniques for interpolation must be considered. The technique briefly described here is block kriging (Burgess and Webster 1980). Block kriging is the form of kriging used to estimate over supports or domains larger than that of the original observations. As such it offers the potential to re-scale data, for example, in a GIS.

An estimate for the larger support B is calculated from a weighted average of known point values (Oliver 1996; Webster 1996):

$$\hat{Z}(B) = \sum_{i=1}^{N} \lambda_i z_v(\mathbf{x}_i) \qquad (6)$$

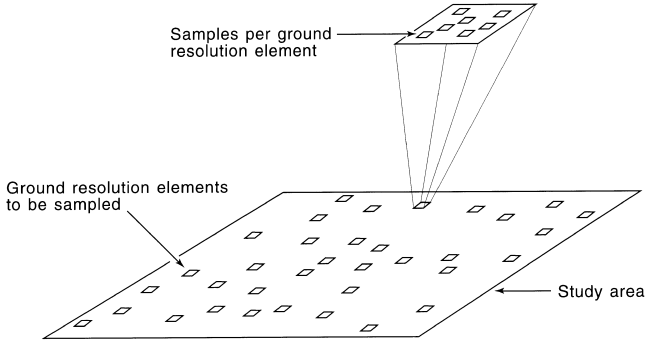

Figure 8: *Diagrammatic representation of the need to sample at the ground within the larger supports of the image pixels to obtain averages which may then be related to the image data (from Curran and Williamson 1985, with kind permission of Taylor and Francis, http://www.tandf.co.uk/journals).*

where the λ_i are the weights. Kriging is statistically unbiased in that the weights in Equation (6) sum to 1, and is optimal in the sense that the weights are chosen to minimize the estimation variance given by:

$$\hat{\sigma}_e^2(B) = \mathrm{E}[\{Z(B) - Z(B)\}^2] \quad (7)$$

which we can re-express in terms of the semivariance to give:

$$\hat{\sigma}_e^2(B) = 2\sum_{i=1}^{N} \lambda_i \bar{\gamma}(x_i, B) - \sum_{i=1}^{N}\sum_{j=1}^{N} \lambda_i \lambda_j \gamma(x_i, x_j) - \bar{\gamma}(B, B) \quad (8)$$

where $\gamma(x_i,x_j)$ is the semivariance between sampling points x_i and x_j, $\bar{\gamma}(x_i,B)$ is the average semivariance between the ith sampling point and the larger cell B, and $\bar{\gamma}(B,B)$ is the average semivariance within B (Oliver 1996; Webster and Oliver 1990). The term on the left of Equation (8) is minimized when

$$\sum_{i=1}^{N} \lambda_i \bar{\gamma}(x_i, x_j) + \psi = \bar{\gamma}(x_i, B) \quad \text{for all } j \quad (9)$$

where the parameter ψ is the Lagrange multiplier required for minimization. Solving the above equations to obtain the weights λ_i is the basis of kriging where $\gamma(x_i,x_j)$, $\bar{\gamma}(x_i,B)$ and $\bar{\gamma}(B,B)$ can be obtained from the variogram model representing the sample data. Block kriging is optimal (in a linear sense) because it accounts for the form and scale of spatial dependence in the property of interest as represented with the variogram. Further, block kriging automatically accounts for the size of support of the estimate. Block kriging may, therefore, seem an ideal choice for data re-scaling.

A major problem is that kriging involves smoothing, where the term "smoothing" is used to mean averaging independent of that which occurs naturally through integration over the support. Smoothing occurs because information from neighboring observations is used in the averaging process, effectively extending the support out to these data. Thus, problems occur when the estimated values are related to a second variable, such as the remotely sensed imagery. As a result of smoothing, the cdf of the ground variable and the bivariate distribution function (bdf) with the remotely sensed variable will be altered: the cdf will have a reduced variance in relation to the true cdf defined on a support of 1.1 km, and the bdf will reflect this change. One suggested solution to the problem of smoothing discussed above involves regularizing the variogram. Regularization is discussed in the next section, and we return to the problem of smoothing in the discussion.

Scaling the Model

Most readers will be familiar with the kriging described above. The geostatistical operation of regularization, described in this section, allows us also to scale the RF model rather than the data. Geostatistical regularization is important because it provides a succinct model of the *process* of re-scaling data.

The sample variogram of some variable is itself defined for a support of given positive (that is, non-zero) size. If it were measured on a different size of support, it would have a different form and would imply a different set of scales of spatial variation. Therefore, it is important to know the support for which the variogram is defined.

Two sets of variograms that were computed from airborne multispectral scanner system (MSS) data in wavebands in the red (0.63–0.69 μm) (Fig. 9a) and near-infrared (0.76–0.90 μm) (Fig. 9b) wavelengths, and at two spatial resolutions of 1.5 m (upper curves) and 2 m (lower curves) (Atkinson 1993) are shown in Figures 9a and 9b respectively. The variograms are fitted with the exponential model. The effect of coarsening the spatial resolution is to remove short-range variation from the variograms so that the semivariance generally decreases. The variograms representing a spatial resolution of 2 m then describe the longer-range variation that remains.

It would be very useful if the effect of the support on the variogram (and, therefore, on measurable scales of variation) could be modelled. Fortunately, the effect of imposing the support of the sampling frame on the underlying variation can be modelled using the variogram. The relation between the punctual or "point" semivariance and the regularized (defined on a support of positive size) semivariance at a lag $\mathbf{h}$ is given (Journel and Huijbregts 1978) by:

$$\gamma_v(\mathbf{h}) = \bar{\gamma}(v, v_{\mathbf{h}}) - \bar{\gamma}(v, v) \quad (10)$$

where $\bar{\gamma}(v,v_{\mathbf{h}})$ is the integral of the punctual semivariance between two supports of size $|\, v \,|$

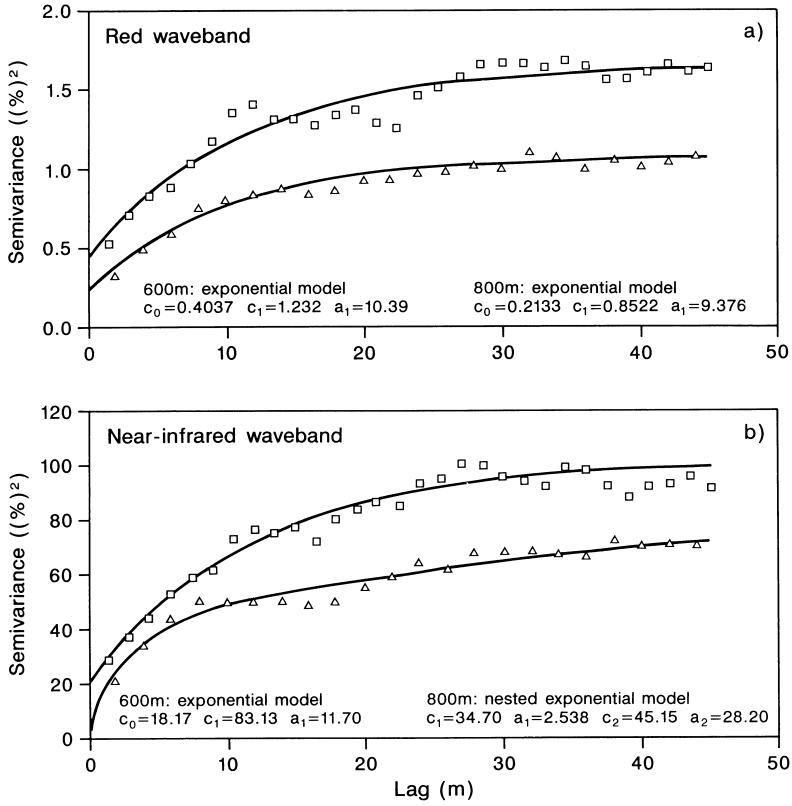

Figure 9: *Sample variograms for the (a) red and (b) near-infrared wavebands obtained at spatial resolutions of 1.5 m (upper curves) and 2 m (lower curves) (from Atkinson 1993, with kind permission of Taylor and Francis, http://www.tandf.co.uk/journals).*

whose centroids are separated by **h**, given formally by

$$\bar{\gamma}(v,v_{\mathbf{h}}) = \frac{1}{v^2}\int_{v}\int_{v(\mathbf{h})} \gamma(\mathbf{y},\mathbf{y}')d\mathbf{y}d\mathbf{y}' \qquad (11)$$

where **y** describes an observation of size $|\ v\ |$ and **y**′ describes independently another observation of equal size and shape at a lag **h** away. The quantity $\bar{\gamma}(v,v)$ is the average punctual semivariance within an observation of size $|\ v\ |$ and is written formally as

$$\bar{\gamma}(v,v) = \frac{1}{v^2}\int_{v}\int_{v} \gamma(\mathbf{y},\mathbf{y}')d\mathbf{y}d\mathbf{y}' \qquad (12)$$

where **y** and **y**′ now cover the same pixel independently.

Given Equation (10), it should be possible to regularize the variogram—that is, increase the size of support on which it is defined—to new larger sizes of support without measuring on that new support. The support must be approximated discretely to allow the integration of the semivariance between cells. The method provides estimates of the regularized semivariance at several discrete lags to which a mathematical model may then be fitted. Thus, the geostatistical operation of regularization embodied in Equation (10) provides an appropriate model for the re-scaling of spatial variation. Since no measurement is required except on the original support, it amounts to scaling the model rather than the data.

In certain circumstances—for example, for core samples in soil survey—the sample support may be so small in relation to the new sup-

port that Equation (10) may be used directly. In most circumstances (for example, where the spatial resolution of remotely sensed imagery is the coarsest possible for the particular sampling density) it may be necessary first to estimate the punctual or point variogram by deregularization. Atkinson (1995) validated such an approach using airborne multispectral imagery obtained at six different spatial resolutions.

As an example, the modelled variogram in Figure 6 (defined for a pixel size of 30 m by 30 m) was deregularized to estimate the punctual variogram (that for a point support) by trial and error. The procedure is different to that of Atkinson (1995) and Atkinson and Curran (1995) and is as follows. First, an exponential model was chosen for the punctual variogram. Next, a value was chosen for the non-linear parameter. With this parameter fixed, the Splus™ code was used to fit iteratively the sill of the exponential model using Equation (10). An initial sill value was provided by the user, the punctual model was regularized to obtain estimates for lags between 300 m and 750 m and the average difference for these 15 lags was used to estimate a new sill as $\{c_{\text{new}} = c_{\text{old}} + \text{avg}(\gamma_{v(obs)}(\mathbf{h}) - \gamma_{v(est)}(\mathbf{h}))\}$. The process was repeated (about five iterations were usually sufficient) with the most recently estimated sill as input for the next iteration until a satisfactory fit was obtained. Visual inspection of the fitted models allowed the whole process to be repeated with new estimates for the non-linear parameters of the models. The resulting punctual model and its coefficients are shown in Figure 10, together with the original sample vanogram (discrete symbols) and the regularized estimates obtained from the punctual models (lower dashed curves).

Once a satisfactory punctual model was fitted, it was possible to regularize the punctual variogram to any desired pixel size using Equation (10). The variogram was regularized to pixel sizes of 80 m by 80 m and 260 m by 260 m: 80 m represents approximately the pixel size of Landsat MSS imagery and 260 m represents that of the delete Medium Resolution Imaging Spectrometer (MERIS) sensor. These regularized variograms are shown along with the punctual models from which they were obtained as the two lower dashed curves in Figure 11. Clearly, the variation one would observe in MERIS data is very different to that observable in Landsat TM and Landsat MSS imagery. This has implications for the kinds of analysis and the set of techniques that might be applied to the new imagery.

In summary, the relationship in Equation (10) provides a means by which to assess the effect of size, shape, and orientation of support on the nature and scale of measured spatial variation (Clark 1977; Jupp et al. 1988, 1989; Atkinson 1993). As such, Equation (10) sup-

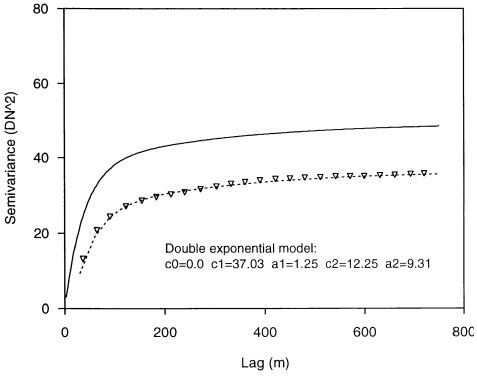

Figure 10: *Sample variogram of Landsat TM image for the red waveband at a pixel size of 30 m fitted with a punctual model (upper solid curve) and regularized to the original pixel size of 30 m (lower dashed curve) (from Curran and Atkinson 1999, with kind permission of Kluwer Academic Publishers).*

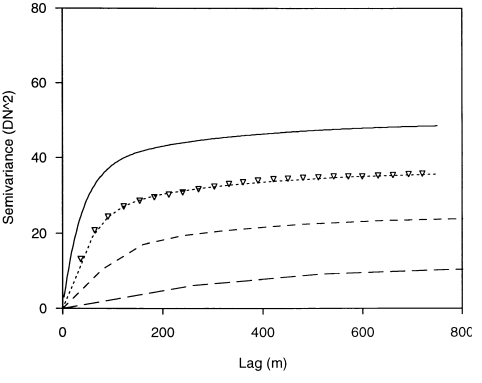

Figure 11: *Sample variogram of Landsat TM image for the red waveband at a pixel size of 30 m fitted with a punctual model (upper solid curve) and regularized to pixel sizes of 80 m (middle dashed curve) and 260 m (lower dashed curve) (from Curran and Atkinson 1999, with kind permission of Kluwer Academic Publishers).*

ports the conceptual model embodied in Figure 1. Equation (10) implies that the only spatial variation detectable from the sample values (made on a support v) is that described by the term $\bar{\gamma}(v,v_h)$. The variation described by $\bar{\gamma}(v,v)$, the within-block variance, is completely obscured from analysis by integration over the support as described in Equation (1). From Equation (10), the effect of regularizing spatial variation over the support is to remove small-scale (that is, short-range or high frequency) variation in favor of large-scale variation occurring within the spatial extent of the sampling frame. If much of the variation is small-scale in relation to the support, then much variation will be removed. If, on the other hand, most variation is large-scale in relation to the support, then only a small amount of variation will be removed. The importance of Equation (10) has to do with more than understanding the effect of the support and regularizing the variogram. Summary statistics such as the dispersion variance $D^2(v,V)$ and *a priori* variance $D^2(v,\infty)$, and simulations may be produced from the regularized variogram for the new support without actually measuring on that support (Zhang et al. 1990).

Discussion

One suggested solution to the problem of smoothing discussed in the previous section involves regularizing the punctual variogram to estimate the dispersion variance $D^2(v,V)$ for the larger supports (Atkinson and Kelly 1997). The idea is that by subtracting $D^2(v,V)$ for the kriged data (σ^2_b) from that for the original point data (σ^2_a) the variance lost due to smoothing (undesirable) and regularization (desirable) (σ^2_c) can be estimated by subtraction. The variance lost through regularization (σ^2_d) can be estimated from the regularized variogram (e.g., Fig. 11) using a discrete approximation of the within-block variance term on the right side of Equation (10) (Journel and Huijgregts 1978). Then, the variance lost due to smoothing (σ^2_e) can be estimated by subtraction (Atkinson and Kelly 1997). Given the reduction in variance in the target or explanatory variable, one can suggest an adjustment to the slope of the regression b using:

$$b = \frac{SSP_{xy}}{SS_x} \qquad (13)$$

where SSP_{xy} is the sum of the squares of the products between x and y (representing the covariance between x and y) and SS_x is the sum of the squares of x (representing the variance of x) (Webster and Oliver 1990). For example, the slope of the regression could be adjusted by the square root of the scalar change to the variance in the kriged variable since, for a constant correlation between x and y, the covariance term may be expected to vary as the square root of the variance in one variable. However, the major obstacle that remains is that the covariance term SSP_{xy} between x and y is not known for the original data. The covariance (with the second variable) can vary between the original and kriged data in unpredictable ways, depending on the scale (component) of spatial variation that is emphasized by kriging. Conditional simulation or stochastic imaging has been suggested as a potential solution to re-scaling (e.g., Journel 1996), but unfortunately it does not solve the covariance problem.

Re-scaling of data is a fundamental objective in operational GIS (e.g., co-registration of data layers). Therefore, the re-scaling issue represents a serious, but often overlooked problem for GIS users.

Conclusions

Scale is an issue of concern to geographers and environmental scientists with a broad range of subject interests. Researchers are frequently concerned with selecting a scale (or increasingly a range of scales) of measurement which will provide informative and accurate spatial variation. Further, they are increasingly required to change the scale of measurement of one variable to facilitate comparison with another, and the latter objective has been most prominent in remote sensing and GIS. There are no straightforward solutions to these problems; worse, there are many pitfalls. For example, when re-scaling to a larger support, if it is not possible to average the spatial variation *within* the larger support, some form of interpolation may be used to "scale-up" the sampling frame. However, with interpolation generally comes smoothing, and this is likely to alter the subsequent relations with other variables.

We suggest that an important first step in addressing scale-related issues is to adopt a conceptual framework in which the spatial varia-

tion observed in data is a function of both the scales of the underlying variation (or more generally, phenomenon) and the scales of measurement (Fig. 1). It is important to gain knowledge of the structure of the property, for example, by computing the sample variogram (or other structure function such as the covariance function). The sample variogram describes the *scales* of spatial variation observed in sample data. Further, given the modelled sample variogram, the geostatistical operation of regularization (Equation 10) can be used to predict changes in the variable with changes in the scales of measurement. This amounts to scaling the RF model rather than the data.

The variogram is of fundamental importance when making decisions about scales of measurement (e.g., which spatial resolution to adopt) and about changing the scales of measurement of data already acquired. Since Equation (10) is generally applicable, it is of fundamental importance to geography. ■

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PETER M. ATKINSON is Reader in the Department of Geography, University of Southampton, Highfield, Southampton SO17 1BJ UK. E-mail: pma@soton.ac.uk. His research interests are focused on geostatistics, spacial modelling, and remote sensing.

NICHOLAS J. TATE is Lecturer in the Department of Geography, University of Leicester, Leicester LE1 7RH UK. E-mail: n.tate@le.ac.uk. His research interests include fractals and geostatistics, as well as GIS and surface modelling in both physical and human contexts.